SPECTRAL AND SCATTERING FEATURES OF HYPERBOLIC MANIFOLDS

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$$\Lambda \subset \Gamma \subset SO(n,1)$$

ZARISKI DENSE
GEOMETRICALLY FINITE

ARITHMETIC LATTICE

$\Lambda(N)$ = KERNEL OF REDUCTION MOD N.

$$\Lambda(N)\backslash \mathbb{H}^n = X(N) \longrightarrow X = \Lambda\backslash \mathbb{H}^n$$

Δ_X, $\Delta_{X(N)}$ LAPLACE BELTRAMI OPERATORS.

Q. WHERE ARE THE NEW EIGENVALUES ON $X(N)$?

MOTIVATION

Ⓐ. BOURGAIN - GAMBURD - SARNAK AFFINE SIEVE

E.G. $\Lambda \subseteq \Gamma = SL_2(\mathbb{Z})$.

$v \in \mathbb{Z}^2$

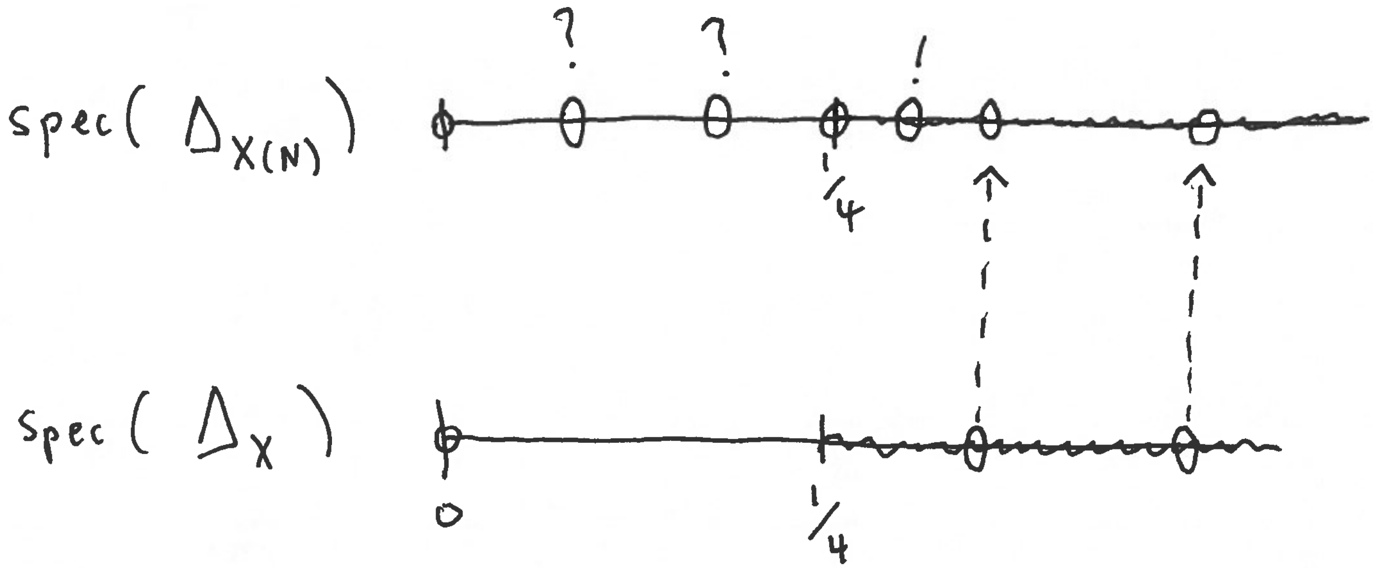
$f \in \mathbb{Z}[x, y]$ w/ SENSIBLE CONDITIONS.

$$f(\Lambda \cdot v) \subseteq \mathbb{Z}.$$

THM (BOURGAIN-GAMBURD-SARNAK)

STRONG UNIFORM SPECTRAL ESTIMATES FOR $X(N)$ GIVE LOWER AND UPPER BOUNDS FOR # OF R-ALMOST PRIMES IN $f(B_\Lambda(0, r) \cdot v)$.

(B) EG $\Lambda = \Gamma = SL_2(\mathbb{Z})$.



~~~~ = ABS. CONTINUOUS

O = DISCRETE

CONJECTURE (SELBERG '65) $\lambda_1(X(N)) \geq 1/4$

THEOREM (SELBERG '65) $\lambda_1(X(N)) \geq 3/16$.

THEOREM (KIM–SARNAK '03) $\lambda_1(X(N)) \geq \dfrac{975}{4096}$.
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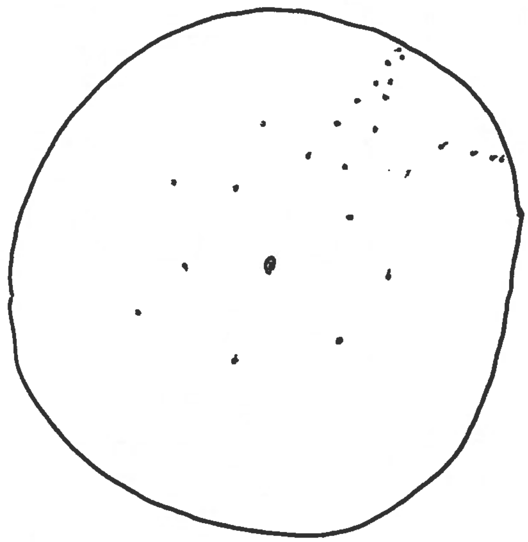
$o \in \mathbb{H}^n$

ORBIT Λo ACCUMULATES (SUBSET OF) ON BOUNDARY S^{n-1}



ACCUMULATION POINTS CALLED LIMIT SET $L(\Lambda)$.

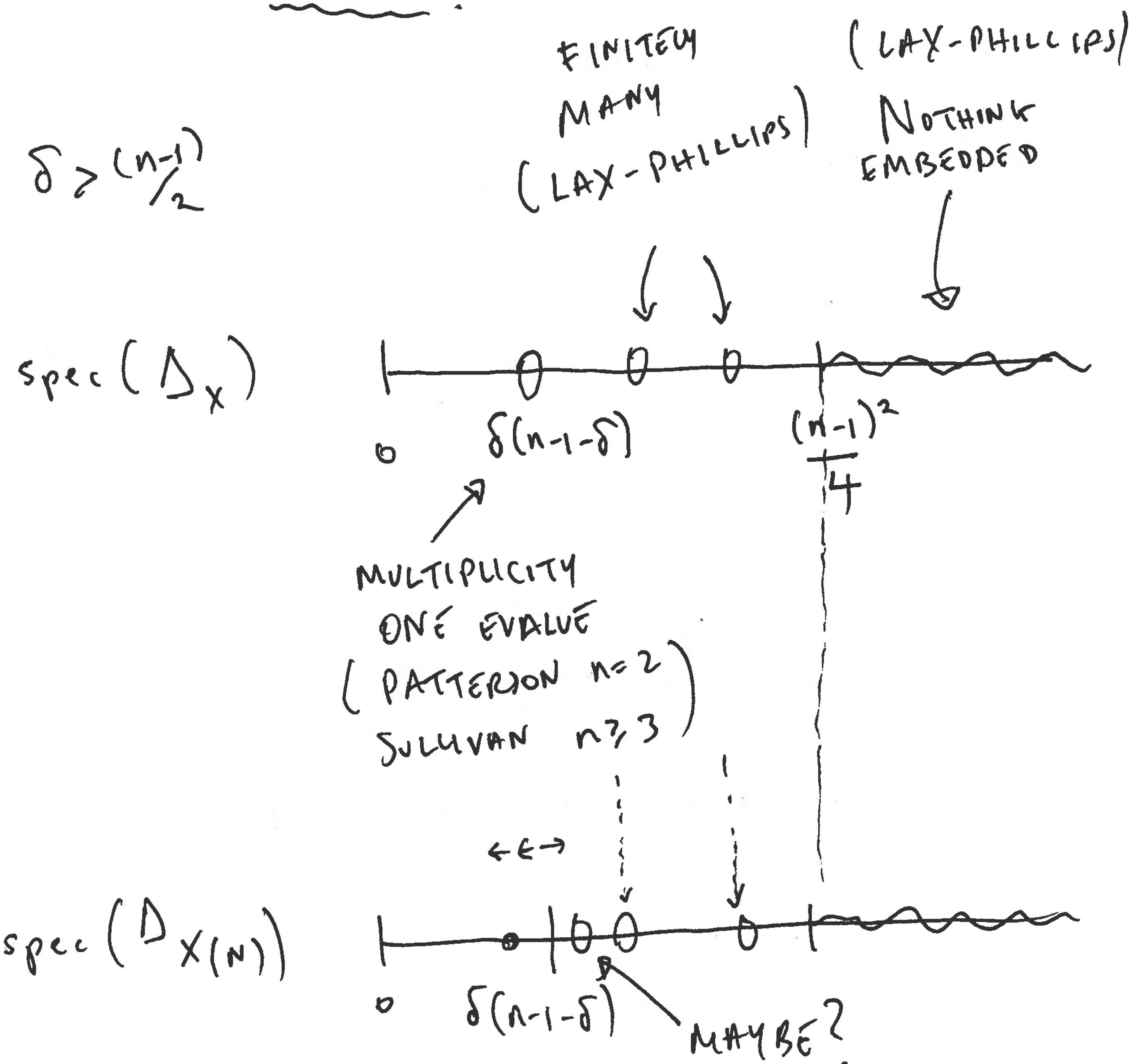
Λ ZARISKI DENSE . INFINITE COVOLUME

$$\Rightarrow \quad \delta = \dim_{\text{HAUSDORFF}} \big(L(\Lambda) \big)$$

$$0 < \delta < n-1.$$

INFINITE VOLUME SPECTRAL THEORY

IF $\delta > (n-1)/2$



THM GAMBURD $n=2$
M $n \geq 3$

WHEN $\delta \geq \delta_0 = \begin{cases} 5/6 & n=2 \\ n-1-\dfrac{2(n-2)}{n(n+1)} & \end{cases}$

$$\operatorname{spec}(\Delta_{X(N)}) \cap [\delta(n-1-\delta), \delta_0(n-1-\delta_0)) = \operatorname{spec}(\Delta_X) \cap [\delta(n-1-\delta), \delta_0(n-1-\delta_0)).$$

TECHNIQUES TO CONTROL SPECTRUM.

(A) TRACE FORMULA. $\displaystyle\sum_{\lambda \text{ EIGENVALUE}} f(\lambda) =$ SOMETHING GEOMETRIC.

- AS IS ONLY GIVES BOUNDS FOR MULTIPLICITIES. (de George - WALLACH).

- COUPLED WITH

KEY FACT THAT.

IDEA OF ;ARNAK -XUE

i) KNOW DECK TRANSFORMATION GROUP BY STRONG APPROXIMATION.

ii) FINITE SIMPLE CHEVALLEY GROUPS HAVE NO SMALL DIM. IRREPS.

(GOWERS - QUASIRANDOMNESS).

TECHNIQUES.

(B) TRANSFER.

(ORIGINATES WITH BROOKS

GOOD EXAMPLE IN
(BOURGAIN - GAMBURD - SARNAK
ACTA '13).

IDEA THAT. $X(N)$

COMBINATORIALLY MODELLED BY

CAYLEY - GRAPH $G(N)$.

BUILT FROM SIDE PAIRING CONGRUENCES
FOR X

AND DECK GROUP $\Lambda / \Lambda(N)$ ← KNOW THIS.

TRY : EXPANSION FOR $G(N)$

$\Rightarrow$ SPECTRAL GAP FOR $X(N)$.

FUTURE DIRECTIONS

$G = SO(n,1)$
$K = SO(n)$.

(A)

$$\begin{tikzcd} & G \arrow[dl] \arrow[dr] & \\ G/K \arrow[dr, "\Lambda"'] & & \Lambda \backslash G \arrow[dl, "K"] \\ & X. & \end{tikzcd}$$

FOR EACH $\tau \in \hat{K}$, HAVE BUNDLE

$E_\tau \to X$. HAS CANONICAL CONNECTION AND LAPLACIAN.

(AMBITIOUS)

(B) CONTROL OF LOW ENERGY ONE FORMS IN COVERING SPACE ASPECT.