Act globally,
Compute locally

Group actions, fixed points and localization

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IAS Members’ Seminar
20 October 2014
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Outline

- Symplectic geometry (with lots of examples)
- Group actions & fixed points (with lots of examples)
- Localization (with lots of examples)
- Symplectic reduction (how to take a quotient) (with lots of examples)
A symplectic manifold is a manifold with a two-form $\omega \in \Omega^2(M)$ that is:

- Closed: $d\omega = 0$
- Non-degenerate: $\omega^n = d\text{Vol} \Rightarrow M$ is $2n$-dimensional & orientable
Symplectic manifolds

Calque of “complex” introduced by Weyl (1939)

Complex: Latin com-plexus “braided together”
Symplectic: Greek συμ - πλεκτικός

\[(\mathbb{R}^2, \omega = dx \wedge dy) \sim \rightarrow (\mathbb{R}^{2n}, \omega = \sum dx_i \wedge dy_i)\]

**Darboux’s Theorem:**

We may always choose coordinates \(x_1, \ldots, x_n, y_1, \ldots, y_n\) on \(M\) so that locally

\[\omega = \sum dx_i \wedge dy_i.\]

\[\sim \rightarrow \text{There are no local invariants (like curvature).}\]
Compact examples

- Even-dimensional spheres \( S^{2n} = \{ \overline{x} \in \mathbb{R}^{2n+1} \mid \sum x_i^2 = 1 \} \)
- Complex projective space \( \mathbb{CP}^{n-1} = \{ V \subseteq \mathbb{C}^n \mid \dim_{\mathbb{C}}(V) = 1 \} \)
- Grassmannian \( \mathcal{G}r(k, \mathbb{C}^n) = \{ V \subseteq \mathbb{C}^n \mid \dim_{\mathbb{C}}(V) = k \} \)
- Flag varieties
  \( \mathcal{F}lags(\mathbb{C}^n) = \{ V_0 \subseteq V_1 \subseteq \cdots \subseteq V_n = \mathbb{C}^n \mid \dim_{\mathbb{C}}(V_i) = i \} \)
- Smooth complex projective varieties
- Toric varieties
- Based loops \( \Omega G = \{ \gamma : S^1 \to G \mid \gamma(\text{Id}) = \text{Id} \} \)
**Example:** $\mathcal{P}ol_d(a_1, \ldots, a_n)$

$\mathcal{P}ol_d(a_1, \ldots, a_n) = \left\{ (\vec{v}_1, \ldots, \vec{v}_n) \mid \vec{v}_i \in \mathbb{R}^d, |\vec{v}_i| = a_i, \sum \vec{v}_i = \vec{0} \right\} / SO(d)$

$\mathcal{P}ol_3(a_1, \ldots, a_n)$ is symplectic! N.B. $d=3$!!
Symplectic actions

Symplectic manifolds often exhibit symmetries, encoded by a group action. (It’s a hard topological question, “How many manifolds do or do not have symmetries?” ...)

DEF: A group action $G \curvearrowright M$ is **symplectic** if it preserves $\omega$; that is,

\[ \tau_g^* \omega = \omega \quad \forall \ g \in G. \]

$S^1 \curvearrowright S^2$ by rotation

$S^1 \curvearrowright T^2$ by rotation
Symplectic manifolds often exhibit symmetries, encoded by a group action. (It’s a hard topological question, “How many manifolds do or do not have symmetries?” ...)

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$$\tau_g^* \omega = \omega \quad \forall g \in G.$$
Symplectic actions

**DEF:** A group action $G \acts M$ is **symplectic** if it preserves $\omega$; that is,

$$\tau_g^* \omega = \omega \quad \forall g \in G.$$ 

**DEF:** Let $G$ be a Lie group with Lie algebra $\mathfrak{g}$. Suppose $G \acts M$. For any $\xi \in \mathfrak{g}$, we may define a **vector field** on $M$ by,

$$\mathcal{X}_\xi(p) = \left. \frac{d}{dt} \left[ \exp(t\xi) \cdot p \right] \right|_{t=0}.$$ 

Infinitesimally

$$\mathcal{L}_{\mathcal{X}_\xi} \omega = 0$$

$$\mathcal{L}_{\mathcal{X}_\xi} \omega = d\iota_{\mathcal{X}_\xi} \omega + \iota_{\mathcal{X}_\xi} d\omega$$

$$\implies d\left( \omega(\mathcal{X}_\xi, \cdot) \right) = 0$$

$S^1 \acts S^2$ by rotation $\xrightarrow{\sim}$ Vector field parallel to latitude lines
Hamiltonian actions

\[ d\left( \omega(X_\xi, \cdot) \right) = 0 \]

**DEF:** Suppose \( G \subset (M, \omega) \). We say the action is Hamiltonian if

\[ \omega(X_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g} \]

**Example:** \( S^1 \subset M = S^2 = \mathbb{C}P^1 \)

\[
\begin{align*}
\omega & = d\theta \wedge dh \\
X_\xi & = \frac{\partial}{\partial \theta} \\
\omega(X_\xi, \cdot) & = dh
\end{align*}
\]

\[ \Rightarrow \quad \phi^\xi = h \]
DEF: Suppose $G \subseteq (M, \omega)$. We say the action is \textit{Hamiltonian} if
\[
\omega(\mathcal{X}_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g}
\]

Non-Example: $S^1 \subseteq T^2 = S^1 \times S^1$ rotating the first factor.

\[
\begin{align*}
\omega & = \ dx \land dy \\
\mathcal{X}_\xi & = \frac{\partial}{\partial x} \\
\omega(\mathcal{X}_\xi, \cdot) & = \ dy
\end{align*}
\]

But $dy \in H^1(T^2; \mathbb{Z})$ is certainly not exact!
Hamiltonian actions

DEF: Suppose $G \subset (M, \omega)$. We say the action is Hamiltonian if

$$\omega(X_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g}$$

Frankel’s Theorem:
A symplectic circle action $S^1 \subset (M, \omega)$ which preserves the compatible complex structure on a compact Kähler manifold $M$ is Hamiltonian $\iff M^{S^1} \neq \emptyset$. 
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Example: $S^1 \subset M = S^2 = \mathbb{C}P^1$

Non-Example: $S^1 \subset T^2 = S^1 \times S^1$

rotating the first factor.
**Frankel’s Theorem:**

A symplectic circle action $S^1 \subset (M, \omega)$ which preserves the compatible complex structure on a compact Kähler manifold $M$ is Hamiltonian

\[ \Leftrightarrow M^{S^1} \neq \emptyset. \]

$T^2 \subset Pol_3(a_1, \ldots, a_5)$ is Hamiltonian.
Frankel’s Theorem: A symplectic circle action $S^1 \subset (M, \omega)$ which preserves the compatible complex structure on a compact Kähler manifold $M$ is Hamiltonian $\iff M^{S^1} \neq \emptyset$.

McDuff’s Theorem: $M$ is compact.

(a) $S^1 \subset (M^4, \omega) \implies$ Hamiltonian $\iff M^{S^1} \neq \emptyset$.

(b) $\exists S^1 \subset (M^6, \omega)$ that has fixed points but is not Hamiltonian.

Questions:
- Are there examples of (b) where the fixed points are isolated?
- Can circle-valued $\phi^\xi$ play an analogous role?
**The momentum map**

**DEF:** Suppose $G \subseteq C(M, \omega)$. We say the action is **Hamiltonian** if

$$\omega(\mathcal{X}_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in g$$

**DEF:** Combining these for all $\xi \in g$, we define the **momentum map**

$$\Phi : M \rightarrow g^*$$

$$p \mapsto \begin{pmatrix} g \\ \xi \end{pmatrix} \mapsto \begin{pmatrix} \mathbb{R} \\ \phi^\xi(p) \end{pmatrix}.$$ 

**Convexity Theorem** [Atiyah, Guillemin-Sternberg]:
If $T = (S^1)^d \subseteq C(M, \omega)$ is Hamiltonian, $\Phi(M)$ is a convex polytope.

$$\Phi(M) = \text{Conv}(\Phi(M^T)).$$
Examples

\[ \mathbb{CP}^3 \]

\[ \text{Gr}(2, \mathbb{C}^4) \]

\[ \mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^1 \]

\[ \mathcal{P}_{\text{ol}}^3(a_1, \ldots, a_5) \]

\[ \mathbb{CP}^2 \]

\[ \mathbb{CP}^1 \times \mathbb{CP}^1 \]

\[ \Omega \text{SU}(2) \]
Localization

A localization phenomenon is a global feature of $T\mathcal{C}M$ that can be described by the evidence of that feature at the $T$-fixed points.

**Convexity Theorem** [Atiyah,Guillemin-Sternberg]:
If $T = (S^1)^d \mathcal{C}(M, \omega)$ is Hamiltonian, $\Phi(M)$ is a convex polytope.

$$\Phi(M) = \text{Conv}(\Phi(M^T)).$$

In terms of topology, we use localization to make global equivariant computations in terms of local computations at fixed points.
Equivariant cohomology is a generalized cohomology theory in the equivariant category.

Functor  $\text{Spaces} \longrightarrow \text{Rings}$
Equivariant cohomology is a generalized cohomology theory in the equivariant category.

 Functor  \[ \text{Spaces} \rightarrow \text{Rings} \]

\[ G \mathcal{O} M \xrightarrow{\sim} H_G^*(M; \mathbb{Z}) \text{ or } H_G^*(M; \mathbb{R}) \]

\[ f : M \rightarrow N \quad \Rightarrow \quad f^* : H_G^*(N) \rightarrow H_G^*(M) \]

Mayer-Vietoris

\[ Et \ cetera \]
Equivariant cohomology is a generalized cohomology theory in the equivariant category.

Functor \( \text{Spaces} \rightarrow \text{Rings} \)

Equivariant cohomology of a point is not \( \mathbb{Z} \)
Equivariant cohomology is a generalized cohomology theory in the equivariant category.

- **Functor** $\text{Spaces} \rightarrow \text{Rings}$

- Equivariant cohomology of a point is not $\mathbb{Z}$

  $$T = T^d = S^1 \times \cdots \times S^1 \quad \Rightarrow$$

  $$H_T^*(\text{pt}; \mathbb{Z}) = \mathbb{Z}[x_1, \ldots, x_d]$$

  $$\deg(x_i) = 2$$
Equivariant cohomology is a generalized cohomology theory in the equivariant category.

- Functor \( \text{Spaces} \to \text{Rings} \)
- Equivariant cohomology of a point is not \( \mathbb{Z} \)
- Spaces, maps should be equivariant
Equivariant cohomology is a generalized cohomology theory in the equivariant category.

Functor \( \mathcal{S} \text{paces} \longrightarrow \mathcal{R}\text{ings} \)

Equivariant cohomology of a point is not \( \mathbb{Z} \)

Spaces, maps should be equivariant

\[
G \overset{\sim}{\leftarrow} M \overset{\sim}{\longrightarrow} H^*_G(M; \mathbb{Z}) \text{ or } H^*_G(M; \mathbb{R})
\]

\( f : M \rightarrow N \implies f^* : H^*_G(N) \rightarrow H^*_G(M) \)

Mayer-Vietoris

\textit{Et cetera}
Equivariant cohomology is a generalized cohomology theory in the equivariant category.

Functor $\text{Spaces} \rightarrow \text{Rings}$

Equivariant cohomology of a point is not $\mathbb{Z}$

Spaces, maps should be equivariant

If $G \subseteq M$ is a free action, then $H^*_G(M) = H^*(M/G)$

Abelianization trick: $H^*_G(M; \mathbb{Q}) \cong H^*_T(M; \mathbb{Q})^W$

(False over $\mathbb{Z}$ -- joint work with Sjamaar)
**Theorem** [Frankel; Atiyah; Kirwan]:
If $T \subset M$ is a compact Hamiltonian $T$-manifold, then
\[ H_T^*(M; \mathbb{Q}) \to H_T^*(M^T; \mathbb{Q}) \]
is an injection. (The statement sometimes holds over $\mathbb{Z}$.)
Example

\[ T^3 \cong Gr(2, \mathbb{C}^4) \sim H^*_T(Gr(2, \mathbb{C}^4); \mathbb{Z}) \subseteq \bigoplus_{i=1}^{6} \mathbb{Z}[x, y, z] \]

\[ \alpha = [\mathbb{P}^2] \]

\[ (z + x)(z + y) \]

\[ (x + y)(x + z) \]

\[ (y + x)(y + z) \]
Equivariant cohomology

\[
\begin{align*}
M^T \hookrightarrow M & \quad \sim \quad H^*_T(M; R) \longrightarrow H^*_T(M^T; R)
\end{align*}
\]


\[
\alpha \in H^*_T(M; R) \quad \iff \quad (\alpha|_N, \alpha|_S) \in \mathbb{R}[x] \oplus \mathbb{R}[x]
\]

**Fact:** \((\alpha|_N, \alpha|_S) \in \text{Im} \iff \chi \mid (\alpha|_N - \alpha|_S)

**GKM Theorem:** Suppose that

(a) \(M^T\) consists of isolated points; and

(b) \(M^S\) consists of isolated points and \(S^2\)s, for each \(S \subset T\) of codimension 1.

Then \(H^*_T(M; \mathbb{Q}) \cong \left\{ (f_v) \in \bigoplus_{v \in M^T} \mathbb{Q}[x_1, \ldots, x_d] \mid \alpha_e | f_v - f_w \text{ for each } S^2_e \right\} \).
Equivariant cohomology

\[ M^{T \subset} \longrightarrow M \overset{\sim}{\longrightarrow} H^*_T(M; R) \longrightarrow H^*_T(M^T; R) \]

The equivariant Chow rings of Quot schemes.

From moment graphs to intersection cohomology.

Ring structures of rational equivariant cohomology rings and ring homomorphisms.

Koszul duality and equivariant cohomology for tori.

Assignments and abstract moment maps.

Variations on themes of Kostant.

Equivariant Schubert calculus.

Koszul duality and equivariant cohomology.

Torsion in equivariant cohomology and Cohen-Macaulay rings.

Combinatorial intersection cohomology.

Rational equivariant cohomology and ring homomorphisms.

Recollections, minor corrections, and computations.

Torsion in the full orbifold cohomology of a graph.

The equivariant cohomology of a graph.

Equivariant quantum Schubert calculus.

Equivariant formality for actions of torus groups.

Equivariant cohomology of type A Springer fibers.

Canonical bases and a $K$-theory Schubert calculus of the affine flag variety.

Weights in the cohomology of toric varieties.

Equivariant cohomology of toric varieties.

Equivariant cohomology of type $A$ Springer fibers.

Equivariant cohomology of Springer fibers.

Equivariant Schubert calculus of the affine flag variety.

The geometric nature of the fundamental lemma.

Weights in equivariant cohomology of type $A$ Springer fibers.

Equivariant cohomology of type $A$ Springer fibers.

Equivariant Schubert calculus of the affine flag variety.

Koszul duality and modular representations of semisimple Lie algebras.


Rebecca Goldin, Megumi Harada, and Tara S. Holm. Torsion in the full orbifold cohomology of a graph.


Milena Hering and Diane Maclagan. The $K$-theory Schubert calculus of the affine flag variety.


Shizuo Kaji. Equivariant Schubert calculus of Coxeter groups.


Victor Guillemin and Catalin Zara. The existence of generating families for the cohomology ring of a graph.


Koszul duality and modular representations of semisimple Lie algebras.


Koszul duality and modular representations of semisimple Lie algebras.


V. Guillemin and C. Zara. Generating families for the cohomology ring of a graph.


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J. Topol. 3 (3) : 661 – 693, 2010.

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Theorem [Harada-Henriques-H.]:
The GKM description works, even in infinite dimensional cases, for

- Equivariant cohomology $H^*_T(M; \mathbb{Q})$
  (Sometimes integrally!) $H^*_T(M; \mathbb{Z})$
- Equivariant K-theory $K^*_T(M)$
- Equivariant cobordism $MU^*_T(M)$
Further applications & generalizations

Schubert calculus

Questions:

- Does this lead to better/easier combinatorics?
- How do you program “subrings” rather than “quotients”?

Quantum invariants and Gromov-Witten theory

Question:

- How do you see quantum corrections in $M^T$?

Torus manifolds (Masuda, Panov, Park)

Toric origami manifolds (H-Pires)

Questions:

- What happens in the non-simply connected case?
- Can you determine manifolds up to cobordism?
Symplectic reduction

We have the moment map

\[ \Phi : M \rightarrow t^* \]

\[ p \mapsto \left( \begin{array}{c} t \\ \xi \end{array} \right) \mapsto \left( \begin{array}{c} t \\ \phi^\xi(p) \end{array} \right). \]

It can be used to prove localization results because \( \phi^\xi \) behaves like a Morse function, with critical set \( M^\Gamma \) (for most \( \xi \)).

The moment map is also an equivariant map: \( T \bigcap \Phi^{-1}(\mu) \) for every \( \mu \in t \). If \( \mu \) is a regular value, \( \Phi^{-1}(\mu) \) is a manifold.

\[
\dim(\Phi^{-1}(\mu)) = \dim(M) - \dim(T) \\
= 2n - d
\]

\[
\dim(\Phi^{-1}(\mu)/T) = \dim(M) - 2 \cdot \dim(T) \\
= 2n - 2d
\]
Symplectic reduction

We have the moment map
\[ \Phi : M \rightarrow \mathfrak{t}^* \]
\[ p \mapsto \begin{pmatrix} \mathfrak{t} \rightarrow \mathbb{R} \\ \xi \mapsto \phi^\xi(p) \end{pmatrix}. \]

It can be used to prove localization results because \( \phi^\xi \) behaves like a Morse function, with critical set \( M^T \) (for most \( \xi \)).

The moment map is also an equivariant map: \( T\mathfrak{c} \Phi^{-1}(\mu) \) for every \( \mu \in \mathfrak{t} \). If \( \mu \) is a regular value, \( \Phi^{-1}(\mu) \) is a manifold.

**Theorem** [Marsden-Weinstein]:
If \( T\mathfrak{c} M \) is a compact Hamiltonian \( T \)-manifold, and \( \mu \) is a regular value of \( \Phi \), then
\[ M//T(\mu) = \Phi^{-1}(\mu)/T \]
is symplectic, with at worst orbifold singularities.
Example of symplectic reduction

\[ S^1 \subset \mathbb{C}^n \]

\[ t \cdot (z_1, \ldots, z_n) = (t \cdot z_1, \ldots, t \cdot z_n) \]

\[ \Phi : \mathbb{C}^n \to \mathbb{R} \]

\[ (z_1, \ldots, z_n) \mapsto \sum |z_i|^2 \]

\[ \Phi^{-1}(\mu) = \left\{ (z_1, \ldots, z_n) \mid \sum |z_i|^2 = \mu \right\} = S^{2n-1} \]

\[ \Phi^{-1}(\mu)/S^1 = S^{2n-1}/S^1 = \mathbb{C}P^{n-1} \]
More examples

Delzant’s Theorem:
\[
\begin{align*}
\{ \text{compact toric symplectic manifolds} \} & \leftrightarrow \{ \text{simple rational smooth convex polytopes} \} \\
\mathbb{C}^n \amalg T^d & \leftrightarrow \mathbb{C}^\text{Flags}(\mathbb{C}^n) \\
T^n \mathbb{C}^\text{Gr}(k, \mathbb{C}^n) & \leftrightarrow \mathcal{O}_\lambda \amalg T \text{ is a weight variety.}
\end{align*}
\]
Yet another example

\[ \mathcal{P}_{\text{ol}_3}(a_1, \ldots, a_n) = \left\{ (v_1, \ldots, v_n) \left| v_i \in \mathbb{R}^3, |v_i| = a_i, \sum v_i = \vec{0} \right\} / \text{SO}(3) \right. \\
= \left. S^2_{a_1} \times \cdots \times S^2_{a_n} / / \text{SO}(3) \right. \\
\]
Kirwan’s Theorem:
If $T \subseteq M$ is a compact Hamiltonian $T$-manifold, then
\[
\kappa_\mu : H^*_T(M; \mathbb{Q}) \longrightarrow H^*_T(\Phi^{-1}(\mu); \mathbb{Q}) \cong H^*(\Phi^{-1}(\mu)/T; \mathbb{Q})
\]
is a surjection (with isomorphism when $\mu$ is a regular value).

Theorem [H-Tolman]:
If $T \subseteq M$ is a compact Hamiltonian $T$-manifold with connected stabilizer subgroups and if $M^T$ is torsion free, then $\kappa_\mu$ is surjective over $\mathbb{Z}$.

Technique: The map $||\Phi||^2$ is minimally degenerate.

Theorem [H-Karshon]:
Minimal degeneracy is a local condition.
Kirwan’s Theorem:
If $T \subseteq M$ is a compact Hamiltonian $T$-manifold, then

$$
\kappa_\mu : H_T^* (M; \mathbb{Q}) \longrightarrow H_T^* (\Phi^{-1}(\mu); \mathbb{Q}) \cong H^* (\Phi^{-1}(\mu)/T; \mathbb{Q})
$$
is a surjection (with isomorphism when $\mu$ is a regular value).

Theorem [Tolman-Weitsman, Goldin]:
The ideal $\ker(\kappa_\mu)$ is computable in terms of localization.

$$
\ker(\kappa_\mu) \hookrightarrow H_T^* (M) \twoheadrightarrow H^* (M/\Gamma T(\mu))
$$
Computing $\ker(\kappa_\mu)$
Computing $\ker(\kappa_\mu)$
Theorem [Goldin-H-Knutson]:
When $M//T$ is an orbifold,

$$\bigoplus_{t \in T} H^*_T(M^t) \longrightarrow H^*_{CR}(M//T)$$

is surjective, with computable kernel.

degree 0 Gromov-Witten invariants
Theorem [Goldin-H-Knutson]:
When $M//T$ is an orbifold,

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is surjective, with computable kernel.

Examples

- Symplectic toric orbifolds
- Weight varieties
Invariants of symplectic reductions that are orbifolds

**Theorem** [Goldin-H-Knutson]:
When $M//T$ is an orbifold,

$$
\bigoplus_{t \in T} H^*_T(M^t) \longrightarrow H^*_{CR}(M//T)
$$

is surjective, with computable kernel.

**Generalizations**

- Equivariant K-theory (with Goldin, Harada, Kimura)
- Explicit computations in K-theory (with Goldin, Harada)
- Preliminary computations of $QH^*(\mathcal{P}ol_3(\bar{a}))$ (with Chen, Taipale; building on Gonzalez, Woodward, Ziltener, et al)
The End

Thank you!!