Exponential-Time Algorithms for NP Problems: Prospects and Limits

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IAS

Oct. 4, 2013

Basic concepts

- NP **problems**: decision problems whose "Yes" instances have short certificates

  (checkable in time polynomial in input length)

- NP-**complete problems**: "hardest" problems in this class.

- Believed not to be solvable in polynomial time. ("P $\neq$ NP")

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Example: Subset Sum

- **INPUT:** integers $a_1, \ldots, a_n, T$ *//each of bitlength $O(n)$*
- **DECIDE:** is there a subset $J \subseteq [n]$ such that $\sum_{j \in J} a_j = T$?

Natural certificate: the set J.

Naïve algorithm: $\sim n^2 \cdot 2^n$ steps.

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"Meet-in-the-middle" algorithm [Horowitz, Sahni '74] :

1. Compute $L :=$ (all possible subsums of $a_1, \ldots, a_{n/2}$);
2. Compute $R :=$ (all possible subsums of $a_{n/2+1}, \ldots, a_n$);
3. **SORT** each of L, R;
4. Check if $T \in L + R$.

Claim

Each step can be performed in $2^{n/2} \cdot \text{poly}(n)$ steps.

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1	4	5	9	13

3	6	11	17	21

T = 20

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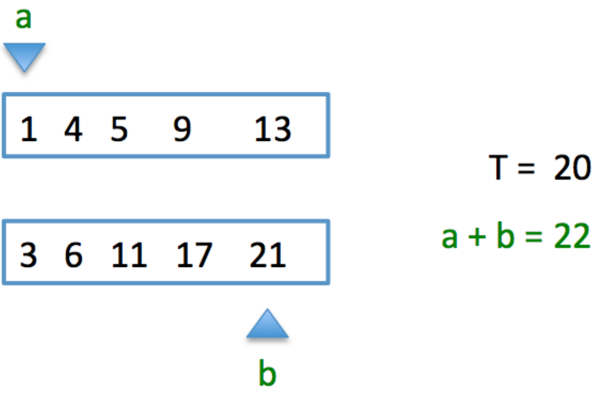

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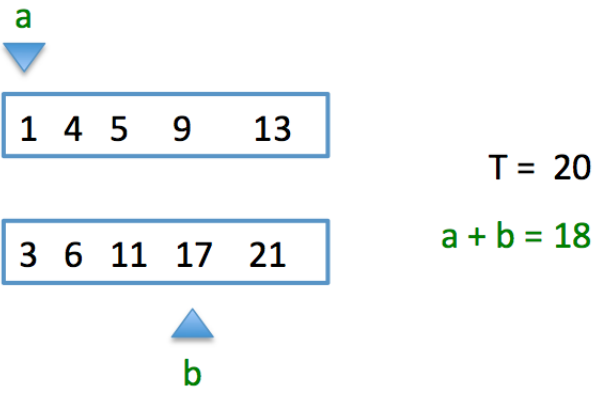

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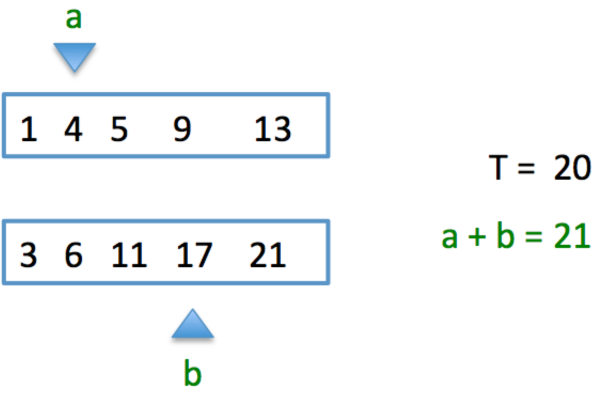

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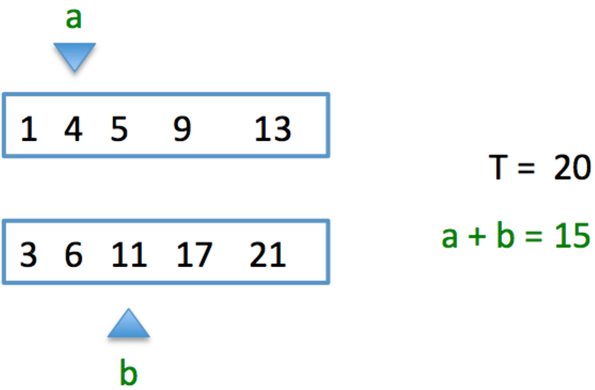

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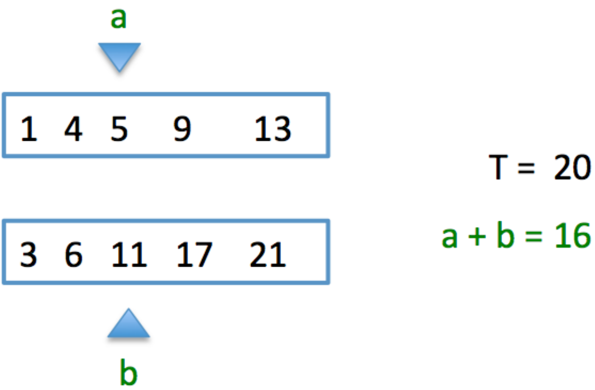

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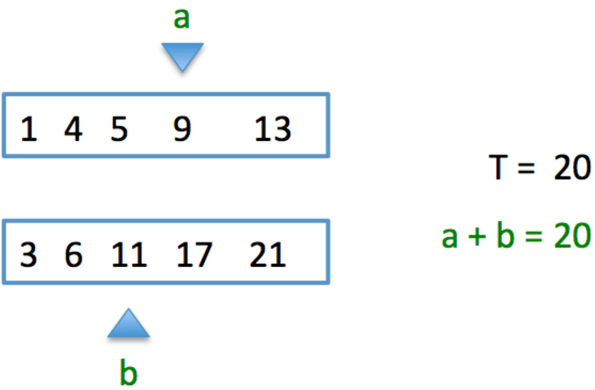

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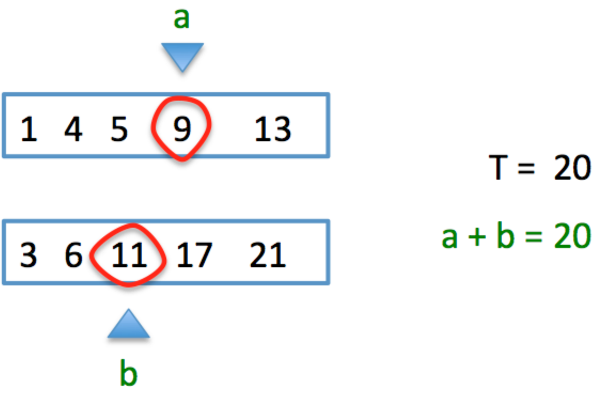

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- $2^{n/2}$ runtime: quite an gain over 2^n...
- **BUT**, since 1974, no further speedups for this problem!

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- No evidence $2^{n/2}$ is optimal!
- One route to progress: the "k-SUM" problem...

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$k \geq 3$ a fixed integer.

- **INPUT:** sets of integers $A_1, \ldots, A_k$ each of size n; a target value T.
- **DECIDE:** Is $T \in A_1 + \ldots + A_k$?
- Best known algorithm: $\sim n^{\lceil k/2 \rceil}$ steps.
  Significant improvements would also improve the best algorithms for SUBSET-SUM and other NP-complete problems.
  OTOH, the ETH implies that k-SUM requires time $n^{\Omega(k)}$.
  [Woeginger '04], [Patrascu, Williams '10]
- Many such connections were found between the complexity of polynomial-time solvable problems (like k-SUM) and NP-complete problems (like SUBSET-SUM).

  Deeper connections may exist.

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For suff. small $\delta > 0$, 3-SAT can't be solved in time $2^{\delta n} \cdot \text{poly}(|\phi|)$.

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Improved algorithm for k-SAT

- Will see a method to solve k-SAT by **intelligent random guessing**.

Theorem [Paturi, Pudlák, Zane '97]

$\exists$ a poly(n)-time randomized algorithm A:
for any satisfiable ϕ, A finds a satisfying assignment with probability $\geq \frac{1}{n} \cdot 2^{-n+n/k}$.

$\implies$ can run A for $\sim n \cdot 2^{n-n/k}$ trials to obtain a solution w.h.p.

- Many of the known improved algs for NP-complete problems have this form! (or, can be re-expressed in this form) [Paturi, Pudlák '10]

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To attempt to produce a satisfying assignment to k-CNF formula $\phi(x_1, \ldots, x_n)$:

Algorithm A [PPZ]:

1. Pick a random permutation $\sigma \in S_n$ ("reordering" of $x_1, \ldots, x_n$);
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  (But, ETH is a very strong assumption...)

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then, Circuit-SAT has (non-uniform) algorithms of runtime $2^{n^{.99}}$.

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What's next?

- May be possible to prove strong limits on other restricted algorithms for solving NP-complete problems.

  (under reasonable hardness assumptions)

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